Uncertainty assessment of effective radiating area and beam non-uniformity ratio of ultrasound transducers determined according to IEC 61689:2007
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Abstract
This work presents the uncertainties in the effective radiating area ($A_{ER}$) and beam non-uniformity ratio ($R_{BN}$) for ultrasound (US) transducers in the range of 1.0 MHz to 3.5 MHz, and for head diameters of 1.27 cm and 2.54 cm. Measurements were performed using the US pressure field mapping system developed at the Laboratory of Ultrasound located at the Brazilian National Metrology Institute, which provides national traceability for the assessed quantities. The calculation protocol was developed based on Standard IEC 61689:2007. The type A uncertainty was estimated from four repetitions of the full measurement procedure for determinations of $A_{ER}$ and $R_{BN}$, and the type B uncertainty was estimated from mathematical models for both parameters, based on IEC 61689:2007 and the ISO ‘Guide to the Expression of Uncertainty in Measurement’. The maximum combined expanded uncertainties (95% confidence level) were 6.8% for $A_{ER}$ and 14.9% for $R_{BN}$.

1. Introduction

Therapeutic ultrasound (TU) is widely used in the frequency range of 0.5 MHz to 3.0 MHz to treat soft tissue lesions, for instance, in the musculoskeletal system [1, 2]. The administration of TU implies the choice of a dose (combination of energy versus time) to achieve clinical results, which are associated with increases in tissue temperature up to healing levels [2, 3]. Intensity levels irradiated through the patient’s body are directly related to temperature. High intensity levels can generate excessive heat, shock waves and cavitation, which can cause permanent damage to biological tissue [4]. Hence, as a preventive standardized measurand, the effective intensity of a physiotherapeutic system, obtained from the quotient of the maximum ultrasonic output power ($P_{out}$) and the effective radiating area ($A_{ER}$), is limited to 3 W cm$^{-2}$ [4].

Another important criterion with regard to safety is the spatial distribution of the ultrasonic beam generated by the therapeutic treatment head. This distribution tends to be non-uniform, as a perfect piston-like transducer is not constructible, and, even if it were, ultrasonic propagation leads to non-uniformities in the near field caused by diffraction due to its finite aperture. Besides, construction details and operation of the treatment head can produce regions of high local pressure. These regions, also called ‘hot spots’, may result in excessive heating in small portions of the tissue, posing the risk of potential harmful effects to the patient.

Non-uniformity can be quantified by the beam non-uniformity ratio ($R_{BN}$), a parameter that represents the ratio of the highest intensity in the field to the average intensity. Based on [4], values of $R_{BN}$ ranging between 3 and 7 are acceptable, whilst transducers presenting $R_{BN} > 8$ are considered unsafe, and it is expected that higher values might cause unwanted biological effects.

Many works have shown the importance of accurately measuring $P_{out}$ and $A_{ER}$ to assess the performance of US physiotherapy equipment [2, 3, 5–7]. However, that has not yet been done for $R_{BN}$. What is more in Brazil,
no statistics are available about the number of ultrasonic physiotherapy treatments carried out, nor whether they are safe or efficient [6, 7]. Moreover, no information is available on the number of pieces of physiotherapy equipment or their working condition [6, 7].

To contribute to an improvement of this situation, the Brazilian National Institute of Metrology, Standardization, and Industrial Quality (Inmetro), through its Laboratory of Ultrasound (Labus), has been making efforts to provide Brazilian traceability for US transducer calibration, US power measurement and US field mapping. The latter procedure is directly related to the scope of this work: measurement of \(A_{ER}\) and \(R_{BN}\), and their respective uncertainties.

According to the ‘Evaluation of Measurement Data—Guide to the Expression of Uncertainty in Measurement—JCGM 100:2008’, the uncertainty of a measurement is defined as a ‘parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand’ [8]. To calculate the uncertainty of a given measurement, it is necessary to take into account two types of uncertainty, namely types A and B. Type A uncertainty is obtained from the statistical analysis of a series of observations. On the other hand, type B comes from sources that cannot be evaluated considering statistical analysis, but can be obtained from previous measurements, knowledge of the behaviour of the measuring equipment, manufacturer’s specifications and data from certificates or handbooks [8].

Reinforcing the importance of careful uncertainty assessment, IEC 61689:2007 stipulates a maximum permissible deviation from the measured to the nominal value for a number of important parameters, including \(A_{ER}\) and \(R_{BN}\).

This work presents the facility for US pressure field mapping, based on current standards. Uncertainties of \(A_{ER}\) and \(R_{BN}\) for US transducers in the range of 1.0 MHz to 3.5 MHz, and diameters of 1.27 cm and 2.54 cm, were assessed to prove the system’s adequacy and reliability.

2. Ultrasonic pressure field mapping system

Labus is equipped with a water bath measuring 1700 mm × 1000 mm × 800 mm, which is large enough for most usual measurements and calibrations in the megahertz frequency range (figure 1(a)). The specified positioning system, used to move the transducer (or hydrophone) in the water bath, allows movement of 300 mm along the X and Y axes, and of 600 mm along the Z axis (Newport Corporation, Irvine, CA, USA) (figure 1(b)). The X and Y axes present resolution and repeatability better than 1.25 \(\mu\)m, whilst Z achieves a maximum of 5.0 \(\mu\)m. Additionally, there is a 360° rotation system, with a resolution of 0.01°. The typical system configuration used during the mapping acquisition comprises a personal computer connected to an oscilloscope, a signal generator and the movement controllers [9].

To integrate all system components, and also to provide a user-friendly interface, a virtual instrument (VI) was developed in LabVIEW (National Instruments Corporation, Austin, TX, USA) [10]. The VI allows control of movement along all axes, the acquisition of waterborne signals and the calculation of essential parameters to assess and calibrate US transducers. In addition, the software automatically performs the raster scans necessary to calculate the parameters related to physiotherapeutic US transducers, based on [4].

The parameters \(A_{ER}\) and \(R_{BN}\) were determined on two transducers of 1.27 cm diameter, and frequencies of 1.0 MHz and 2.25 MHz, and two transducers of 2.54 cm diameter, and frequencies of 1.0 MHz and 3.5 MHz. The transducers are excited using a 20-cycle burst of a sine wave generated by a function generator AFG 3252 (Tektronix, Beaverton, OR, USA), and waterborne signals are acquired using an oscilloscope TDS 3032B (Tektronix, Beaverton, OR, USA). Needle hydrophones are used in the mapping procedure and, for this work, active elements of 0.2 mm and 0.5 mm were applied (Precision Acoustics Ltd, Dorchester, Dorset, UK). These active element diameters were chosen to match the maximum effective radius of the hydrophone (\(a_{max}\)) defined in [4] (\(\lambda/a_{max} \geq 2.5\), where \(\lambda\) is the ultrasonic wavelength). The values of \(a_{max}\) calculated in this work, and the respective hydrophones used to perform the raster scans are presented in table 1.

Transducers are first mapped along the beam axis using steps of 0.1 cm to determine the position of the last maximum of pressure of the transducer beam (\(Z_N\)). The effective radius (\(a_t\)) is then estimated using \(a_t = \sqrt{Z_N \cdot \lambda}\) [11]. The transducers are mapped over two planes, initially at 0.3 cm from the treatment head face and then at \(Z_N\). Both planes are defined with dimensions of 80 mm × 80 mm, again using a fixed step of 0.1 cm.
3. Effective radiating area

The effective radiating area \((A_{\text{ER}})\) of the treatment head is calculated by multiplying the beam cross-sectional area determined at 0.3 cm from the front face of the treatment head and parallel to it, \(A_{\text{BCS}}(0.3)\), by a dimensionless factor, \(F_{\text{ac}}\), as presented in equation (1):

\[
A_{\text{ER}} = A_{\text{BCS}}(0.3) \cdot F_{\text{ac}}. \tag{1}
\]

According to [4], the conversion factor \(F_{\text{ac}} = 1.354\) is used in order to derive the area close to the treatment head which contains 100% of the total mean square acoustic pressure.

The value of each \(A_{\text{BCS}}(0.3)\) is given by \(n \cdot s^2\), where \(s^2\) is the unit area of the raster scan and \(n\) is determined by [4]

\[
1 \frac{M_2^2}{M_1^2} \sum_{i=1}^{N} V_i^2 \leq 0.75 \frac{M_2^2}{M_1^2} \sum_{i=1}^{N} N^2 < \frac{1}{M_1^2} \sum_{i=1}^{N} V_i^2, \tag{2}
\]

where \(V_i^2\) is the peak voltage of the \(i\)th point in the scan, \(N\) is the total number of points in the scan and \(M_2^2\) is the end-of-cable loaded sensitivity of the hydrophone. The \(M_2^2\) value has been introduced for convenience in (2) to convert the measured voltage to acoustic pressure. However, neither its absolute nor its complex values are required, unless phase aspects are of concern [12]. As IEC 61689:2007 makes no reference to the derivative of this expression for the variables \(F_{\text{ac}}, n\) and \(s\).

Hence, the expression of \(A_{\text{ER}}\) can be written as

\[
A_{\text{ER}} = F_{\text{ac}} \cdot n \cdot s^2. \tag{3}
\]

To obtain the uncertainty in \(A_{\text{ER}}\) (see [8] and, further, equation (8), it is thus necessary to calculate the partial derivative of this expression for the variables \(F_{\text{ac}}, n\) and \(s\).

4. Beam non-uniformity ratio

The beam non-uniformity ratio \((R_{\text{BN}})\) is defined as the ratio of the square of the maximum rms acoustic pressure \(P_{\text{max}}\) to the spatial average of the square of the rms acoustic pressure, where the spatial average is taken over the \(A_{\text{ER}}\). According to [4], \(R_{\text{BN}}\) is calculated as

\[
R_{\text{BN}} = \frac{P_{\text{max}}^2 A_{\text{ER}}}{P_{\text{rms}}^2}, \tag{4}
\]

where \(s^2\) is the unit area of the raster scan and \(P_{\text{rms}}\) is the total mean square acoustic pressure. The pressures \(P_{\text{max}}\) and \(P_{\text{rms}}\) are defined as follows:

\[
p_{\text{rms}} = \sum_{i=1}^{N} V_i^2 \quad \text{and} \quad P_{\text{max}} = \frac{V_{\text{max}}^2}{M_2^2}. \tag{5}
\]

The product \(P_{\text{rms}} s^2\) is calculated by averaging the squared-pressure values over the areas of raster scans in the plane at 0.3 cm from the treatment head, and in the plane at \(Z_N\), as presented in equation (6).

\[
p_{\text{rms}} s^2 = \frac{1}{4} \left[ \left( P_{\text{rms}}(0.3) s^2(0.3) \right) + \left( P_{\text{rms}}(Z_N) s^2(Z_N) \right) \right]. \tag{6}
\]

Although \(P_{\text{max}}\) and \(P_{\text{rms}}\) are referred to acoustic pressure or squared-pressure parameters, only their ratio is required for the determination of \(R_{\text{BN}}\), so the end-of-cable loaded sensitivity of the hydrophone is not necessary [4]. Based on this consideration, \(R_{\text{BN}}\) can be expressed as

\[
R_{\text{BN}} = 2 \frac{V_{\text{max}}^2 A_{\text{ER}}}{\left[ P_{\text{rms}}(0.3) s^2(0.3) \right] + \left[ P_{\text{rms}}(Z_N) s^2(Z_N) \right]}, \tag{7}
\]

where \(v_{\text{rms}} = \sum_{i=1}^{N} V_i^2\). In this work, \(s(0.3) = s(Z_N)\).

5. Derivation of the effective radiating area uncertainty model

For a generic relation \(h = f(x_j)\), according to [8], the general formula to be used in the uncertainty model is

\[
u^2_e = \sum_{j=1}^{N} \left( \frac{\partial h}{\partial x_j} \right)^2 \cdot u_j^2, \tag{8}
\]

where \(u_j\) is the combined standard uncertainty associated with the final result of measurement (or calculus) of \(h\), and \(x_j\) is a standard uncertainty, assessed as type A or type B, associated with each parameter or variable \(x_j\) used to express the value of \(h\). This model is to be applied to the equation used to completely define the measurand.

The sensitivity coefficients \(c_x, c_s\) and \(c_{F_{\text{ac}}}\) related to \(n, s\) and \(F_{\text{ac}}\), respectively, are obtained from the partial derivatives of equation (3):

\[
c_x = \frac{\partial A_{\text{ER}}}{\partial x} = 2 \cdot F_{\text{ac}} \cdot n \cdot s, \tag{9}
\]

\[
c_s = \frac{\partial A_{\text{ER}}}{\partial n} = F_{\text{ac}} \cdot s^2, \tag{10}
\]

\[
c_{F_{\text{ac}}} = \frac{\partial A_{\text{ER}}}{\partial F_{\text{ac}}} = n \cdot s^2. \tag{11}
\]

Type A and type B uncertainties for \(n\) \((u_{n\text{(type A)}})\) and \(u_{n\text{(type B)}}\), \(s\) \((u_{s\text{(type A)}})\) and \(u_{s\text{(type B)}}\) and \(F_{\text{ac}}\) \((u_{F_{\text{ac}}\text{(type A)}})\) and \(u_{F_{\text{ac}}\text{(type B)}}\), as well as other sources of uncertainty due to
truncation of raster scan ($u_{\text{truncation}}$), selected step size ($u_{\text{step size}}$) and spatial averaging ($u_{\text{spatial averaging}}$), are defined later within this paper.

The combined uncertainty of $A_{ER}$ can be expressed as follows:

$$u^2_{A_{ER}} = c^2_s (u^2_{\text{type A}}) + u^2_{\text{type B}} + c^2_{\text{dr}} (u^2_{\text{dr}}) + u^2_{\text{noise}} + u^2_{\text{truncation}} + u^2_{\text{step size}} + u^2_{\text{spatial averaging}} + u^2_{\text{uncertainty}}$$

(12)

### 6. Derivation of the beam non-uniformity uncertainty model

The sensitivity coefficients for calculating $R_{\text{BN}}$ uncertainty are obtained from partial derivatives of equation (7):

$$c_{V_{\text{max}}} = \frac{\partial R_{\text{BN}}}{\partial V_{\text{max}}} = 4 \frac{V_{\text{max}}^2 A_{ER}}{(v_{\text{rms}}(0.3) + v_{\text{rms}}(Z_N))^2 \cdot s^2}$$

(13)

$$c_{A_{\text{ER}}} = \frac{\partial R_{\text{BN}}}{\partial A_{\text{ER}}} = 4 \frac{V_{\text{max}}^2 A_{\text{ER}}}{(v_{\text{rms}}(0.3) + v_{\text{rms}}(Z_N))^2 \cdot s^2}$$

(14)

$$c_{v_{\text{rms}}(0.3)} = \frac{\partial R_{\text{BN}}}{\partial v_{\text{rms}}(0.3)} = -2 \frac{V_{\text{max}}^2 A_{\text{ER}}}{(v_{\text{rms}}(0.3) + v_{\text{rms}}(Z_N))^2 \cdot s^2}$$

(15)

$$c_{v_{\text{rms}}(Z_N)} = \frac{\partial R_{\text{BN}}}{\partial v_{\text{rms}}(Z_N)} = -2 \frac{V_{\text{max}}^2 A_{\text{ER}}}{(v_{\text{rms}}(0.3) + v_{\text{rms}}(Z_N))^2 \cdot s^2}$$

(16)

$$c_s = \frac{\partial R_{\text{BN}}}{\partial s} = -4 \frac{V_{\text{max}}^2 A_{\text{ER}}}{(v_{\text{rms}}(0.3) + v_{\text{rms}}(Z_N))^2 \cdot s^2}$$

(17)

Hence, the combined standard uncertainty of $R_{\text{BN}}$ can be expressed as

$$u^2_{R_{\text{BN}}} = c^2_s (u^2_{\text{type A}}) + u^2_{\text{type B}} + c^2_{\text{dr}} (u^2_{\text{dr}}) + u^2_{\text{noise}} + u^2_{\text{truncation}} + u^2_{\text{step size}} + u^2_{\text{spatial averaging}} + u^2_{\text{uncertainty}}$$

(18)

As previously presented, $v_{\text{rms}} = \sum_{i=1}^{N} V_i^2$, so $v_{\text{rms}} = V_1^2 + V_2^2 + \cdots + V_i^2$, where $N$ is the total number of points in the specific plane (0.3 or $Z_N$). To estimate the $v_{\text{rms}}$ uncertainty ($u_{\text{v_{rms}}}$), it is necessary to calculate the sensitivity coefficients by calculating the partial derivative of $v_{\text{rms}}$ as follows:

$$c_{V_1} = \frac{\partial v_{\text{rms}}}{\partial V_1} = 2 V_1, \quad c_{V_2} = \frac{\partial v_{\text{rms}}}{\partial V_2} = 2 V_2, \quad \ldots \quad c_{V_N} = \frac{\partial v_{\text{rms}}}{\partial V_N} = 2 V_N.$$  

(19)

Hence, the uncertainty in $v_{\text{rms}}$ is

$$u_{\text{v_{rms}}} = \sqrt{u^2_{V_1}(\text{type A}) + u^2_{V_2}(\text{type B}) + \cdots + u^2_{V_N}(\text{type B})}.$$  

(20)

### 7. Determination of standard uncertainty of types A and B

According to JCGM 100:2008 [8], an instrument resolution is a type B uncertainty, and is to be assessed taking half of the resolution value as a rectangular distribution, dividing it by $\sqrt{3}$. Herein, the finite resolution of the positioning system used to perform the raster scans is assumed to present a rectangular distribution. Hence, the type B uncertainty of $s (u_{\text{step size}})$ is estimated by dividing the equipment resolution (1.25 x 10^{-4} cm) by $2\sqrt{3}$. The type A uncertainty ($u_{\text{truncation}}$) of the position system is estimated, for each of the three axes, as the standard deviation of the mean of five measurements of their linear translation.

The type B uncertainty for amplitude measurements ($u_{V_i}(\text{type B})$) is estimated as the quadratic combination of three sources of uncertainty. The first one is the oscilloscope precision, defined in its manual as 0.02 · $V_i$ + 0.05 · [vertical scale], while the second is the oscilloscope resolution estimated as $V_i/(512 \cdot \sqrt{2})$ for a 9-bit oscilloscope. Both of them are estimated directly by the VI. The last one is the acquisition system linearity that takes into account the linearity of the combined set hydrophone + amplifier + oscilloscope. As stipulated by IEC 61689:2007, item 6.4, this is estimated by measuring the signal received by the hydrophone and measuring system as a function of the voltage excitation applied to the transducer. Herein, we apply to an ultrasonic transducer, operating in tone-burst mode, peak-to-peak voltage excitations of 0.25 V, 1 V, 2 V, 5 V and 10 V, comprising variation of 32 dB between the lowest and highest amplitude values. Then, the signal generated by the hydrophone is measured with the aid of the oscilloscope. All measurements are performed over the last axial maximum, $Z_N$, and they are carried out with the four hydrophone–transducer pairs. Based on these results, the linear regression for the four pairs is determined and the root mean squared error ($e_{\text{rms}}$) is used as the uncertainty related to the system linearity. The $e_{\text{rms}}$ values obtained for the hydrophone–transducer pairs are presented in table 1. The type A uncertainty for an amplitude measurement ($u_{V_i}(\text{type A})$) is estimated directly by the VI, as the standard deviation of the mean of five measurements, divided by $\sqrt{5}$. Concerning noise contribution in amplitude measurements, it is noted that signals are corrected as described in [4] item B.2.4. In all cases, the signal-to-noise ratio was better than −38 dB. The type A uncertainty of $n (u_{\text{step size}}(\text{type A}))$ is estimated based on the influence of the amplitude uncertainty. The value of $n$ is calculated by adding the amplitude uncertainty to each point of the scan, and the difference between the results, divided by 2, is assumed to be the uncertainty of $n$. An example of that calculation, considering each one of the four repetitions, is presented in table 2.
Uncertainty assessment of effective radiating area and beam non-uniformity ratio of ultrasound transducers

Uncertainty assessment of effective radiating area and beam non-uniformity ratio of ultrasound transducers

Table 3. Example of $A_{ER}$ uncertainty budget for the 1.0 MHz transducer (diameter: 2.54 cm).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Estimation</th>
<th>Standard uncertainty</th>
<th>Probability distribution</th>
<th>Sensitivity coefficients</th>
<th>Uncertainty contribution</th>
<th>Degrees of freedom ($v_i$)</th>
<th>Combined uncertainty (equation (12))</th>
<th>$\nu_{eff}$</th>
<th>Coverage factor (95%)</th>
<th>Expanded uncertainty</th>
<th>cm²</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step/cm $u_{d,type B}$</td>
<td>0.10</td>
<td>$1.25 \times 10^{-4}$</td>
<td>Rectangular</td>
<td>84.5</td>
<td>$3.05 \times 10^{-3}$</td>
<td>$\infty$</td>
<td>$9.15 \times 10^{-2}$</td>
<td>&gt;100</td>
<td>2</td>
<td>$1.9 \times 10^{-1}$</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>Step/cm $u_{d,type A}$</td>
<td>0.10</td>
<td>$1.11 \times 10^{-4}$</td>
<td>Normal</td>
<td>84.5</td>
<td>$4.21 \times 10^{-3}$</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$ dimensionless</td>
<td>312</td>
<td>3.75</td>
<td>Rectangular</td>
<td>1.35 $\times 10^{-2}$</td>
<td>$5.08 \times 10^{-2}$</td>
<td>$\infty$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_{n,type B}$</td>
<td>1.354</td>
<td>0.09</td>
<td>Normal</td>
<td>3.12</td>
<td>$3.46 \times 10^{-2}$</td>
<td>65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Truncation of the raster scan/cm²</td>
<td>4.22</td>
<td>4.22 - 0.6%</td>
<td>Normal</td>
<td>1.0</td>
<td>$2.53 \times 10^{-2}$</td>
<td>$\infty$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scan step size/cm²</td>
<td>4.22</td>
<td>4.22 - 1.0%</td>
<td>Normal</td>
<td>1.0</td>
<td>$4.22 \times 10^{-2}$</td>
<td>$\infty$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spatial averaging/cm²</td>
<td>4.22</td>
<td>4.22 - 1.0%</td>
<td>Normal</td>
<td>1.0</td>
<td>$4.22 \times 10^{-2}$</td>
<td>$\infty$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_{ER}$/cm² $u_{A_{ER},type A}$</td>
<td>4.22</td>
<td>1.89 $\times 10^{-2}$</td>
<td>Normal</td>
<td>1.0</td>
<td>$1.89 \times 10^{-2}$</td>
<td>4</td>
<td></td>
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</tr>
</tbody>
</table>

Taking into account the step size used (0.1 cm), the values of $n$ for the transducer of 2.54 cm diameter ($n > 300$) agree with the IEC 61689:2007 criterion, where the number of points, $n$, included in the determination of $A_{BCS}$, should be at least 100. However, this was not the case for transducers of diameter 1.27 cm ($80 < n < 100$). Hence, an uncertainty of ±1.0% ($u_{step size}$), due to the selected step size, is included in the $A_{ER}$ uncertainty determination for both transducer sizes [13]. Moreover, the uncertainty due to spatial averaging ($u_{spatial averaging}$) is considered to be ±1.0%, also based on [13]. It is of note that spatial-averaging corrections ($c_{A}$) are calculated at the $Z_N$ position for hydrophone–transducer combinations, as defined in [14] (Annex J, item J.2), and their values (table 1) are considered to correct the $V_{max}$ amplitude.

Based on [4], the standard deviation of $F_{ac}$ in the mean value is approximately 0.09, for a sample size of 66 points. Consequently, the type B uncertainty of $F_{ac}$ ($u_{F_{ac},type B}$) is estimated to be $0.09/\sqrt{66}$. Moreover, the uncertainty due to truncation of the raster scan ($u_{truncation}$) is assumed to be ±0.6%, based on [13].

Combination of the above types A and B components gives the overall combined uncertainty of $A_{ER}$ and $R_{BN}$. Calculations are incorporated within the VI, and determined for each one of the four repetitions of a complete procedure. Therefore, the highest value of the combined uncertainties from each one of the four repetitions is combined with the type A uncertainty of the whole process ($u_{A_{ER,type A}}$ or $u_{F_{ac},type A}$), to give the final combined uncertainty of the whole measurement. An example uncertainty budget for $A_{ER}$ and $R_{BN}$ (confidence level of 95%) is presented in tables 3 and 4, respectively.

8. Experimental results

The values of $A_{ER}$ for the transducers of diameter 1.27 cm ranged from 1.12 cm² (2.25 MHz transducer) to 1.19 cm² (1.0 MHz transducer) (table 5). The $A_{ER}$ mean value for the 1 MHz transducer was 1.181 cm², with an estimated expanded uncertainty of $6.3 \times 10^{-2}$ cm² (5.3%), whilst for the 2.25 MHz transducer it was 1.158 cm², with an expanded uncertainty of $7.9 \times 10^{-2}$ cm² (6.8%). The values of $A_{ER}$ varied from 4.18 cm² (1.0 MHz transducer) to 4.71 cm² (3.5 MHz transducer) for transducers of diameter 2.54 cm. The $A_{ER}$ mean value of the 1 MHz transducer was 4.22 cm² (expanded uncertainty $1.9 \times 10^{-1}$ m², 4.5%), whilst for the 3.5 MHz transducer it was 4.70 cm² (expanded uncertainty $1.7 \times 10^{-1}$ cm², 3.6%). Table 6 compares the effective transducer area ($A_{E}$), calculated from the transducer effective radius ($a_{th}$), with the measured values $A_{ER}$. One observes that $(A_{ER} - 2\mu_{A_{ER}}) < A_{E} < (A_{ER} + 2\mu_{A_{ER}})$, and this is discussed below.

Of all the tests performed, with all the transducers studied, the highest value of $R_{BN}$ was 3.57 (3.5 MHz transducer, $\varnothing = 2.54$ cm), whilst the lowest value was 2.71 (2.25 MHz transducer, $\varnothing = 1.27$ cm) (table 7). Consequently, the 2.25 MHz transducer ($\varnothing = 1.27$ cm) presented the lowest mean value of $R_{BN}$ (2.76), and the 3.5 MHz transducer ($\varnothing = 2.54$ cm) furnished the highest one (3.51). For illustrative purpose, the raster scans showing the highest and lowest individual values of $R_{BN}$ are presented in figures 2 and 3, respectively. The lowest uncertainty (12.6%) was achieved using the 1.0 MHz transducer with a 2.54 cm diameter head ($R_{BN} = 3.25 \pm 0.41$), whilst the highest uncertainty (14.9%) was obtained with the 2.25 MHz transducer and 1.27 cm diameter head ($R_{BN} = 2.76 \pm 0.41$) (table 7).

9. Discussion

From the nominal diameters of the transducers studied (1.27 cm and 2.54 cm), their respective nominal areas are approximately 1.27 cm² and 5.07 cm². However, we observe that the effective areas ($A_{E}$) differ from the nominal ones (table 6), and are smaller in most cases. This difference might be associated with aspects of transducer construction, ceramic apodization and other effects. In contrast, the measured effective radiating area ($A_{ER}$) values were in accordance with the calculated $A_{E}$ values, as shown in table 6. These...
Table 4. Example of $R_{BN}$ uncertainty budget for the 1.0 MHz transducer (diameter: 2.54 cm).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Estimation</th>
<th>Standard uncertainty</th>
<th>Probability distribution</th>
<th>Sensitivity coefficients</th>
<th>Uncertainty contribution (equation (9))</th>
<th>Degrees of freedom ($v$)</th>
<th>Combined uncertainty (equation (18)) $v_{eff}$</th>
<th>Coverage factor (95%)</th>
<th>Expanded uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$V, cm$^{-1}$</td>
<td>0.10</td>
<td>$2.50 \times 10^{-3}$</td>
<td>Rectangular</td>
<td>$-66.3$</td>
<td>$-2.39 \times 10^{-3}$</td>
<td>$\infty$</td>
<td>$2.02 \times 10^{-1}$</td>
<td>&gt;100</td>
<td>$4.1 \times 10^{-1}$</td>
</tr>
<tr>
<td>$\Delta$V, cm$^{-1}$</td>
<td>0.10</td>
<td>$1.11 \times 10^{-4}$</td>
<td>Normal</td>
<td>$-66.3$</td>
<td>$-3.30 \times 10^{-3}$</td>
<td>$4$</td>
<td>$2.22 \times 10^{-2}$</td>
<td>$4$</td>
<td>$3.1 \times 10^{-2}$</td>
</tr>
<tr>
<td>$A_{ER}$, cm$^2$ combined</td>
<td>4.22</td>
<td>$9.15 \times 10^{-2}$</td>
<td>Normal</td>
<td>$7.85 \times 10^{-1}$</td>
<td>$7.18 \times 10^{-2}$</td>
<td>&gt;100</td>
<td>$1.85 \times 10^{-1}$</td>
<td>$\infty$</td>
<td>$4.2 \times 10^{-1}$</td>
</tr>
<tr>
<td>$V_{max}$, cm$^{-1}$</td>
<td>$3.47 \times 10^{-2}$</td>
<td>Rectangular</td>
<td>$191$</td>
<td></td>
<td>$1.85 \times 10^{-1}$</td>
<td>$\infty$</td>
<td>$2.02 \times 10^{-1}$</td>
<td>&gt;100</td>
<td>$4.1 \times 10^{-1}$</td>
</tr>
<tr>
<td>$V_{max}$, cm$^{-1}$</td>
<td>$3.47 \times 10^{-2}$</td>
<td>Normal</td>
<td>$191$</td>
<td></td>
<td>$2.22 \times 10^{-2}$</td>
<td>$4$</td>
<td>$3.06 \times 10^{-2}$</td>
<td>$4$</td>
<td>$3.1 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\mu_{V}(0.3)N^2$</td>
<td>$1.56 \times 10^{-1}$</td>
<td>Rectangular</td>
<td>$10.8$</td>
<td></td>
<td>$-5.63 \times 10^{-3}$</td>
<td>$\infty$</td>
<td>$2.02 \times 10^{-1}$</td>
<td>&gt;100</td>
<td>$4.1 \times 10^{-1}$</td>
</tr>
<tr>
<td>$\mu_{V}(0.3)N^2$</td>
<td>$1.56 \times 10^{-1}$</td>
<td>Normal</td>
<td>$10.8$</td>
<td></td>
<td>$-5.38 \times 10^{-3}$</td>
<td>$4$</td>
<td>$2.22 \times 10^{-2}$</td>
<td>$4$</td>
<td>$3.1 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\mu_{V}(2.54)N^2$</td>
<td>$1.51 \times 10^{-1}$</td>
<td>Rectangular</td>
<td>$10.8$</td>
<td></td>
<td>$-5.30 \times 10^{-3}$</td>
<td>$\infty$</td>
<td>$2.22 \times 10^{-2}$</td>
<td>$4$</td>
<td>$3.1 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\mu_{V}(2.54)N^2$</td>
<td>$1.51 \times 10^{-1}$</td>
<td>Normal</td>
<td>$10.8$</td>
<td></td>
<td>$-5.07 \times 10^{-3}$</td>
<td>$4$</td>
<td>$3.06 \times 10^{-2}$</td>
<td>$4$</td>
<td>$3.1 \times 10^{-2}$</td>
</tr>
<tr>
<td>$R_{BN}$, dimensionless</td>
<td>3.25</td>
<td>$3.06 \times 10^{-2}$</td>
<td>Normal</td>
<td></td>
<td>$3.06 \times 10^{-2}$</td>
<td>$4$</td>
<td>$3.06 \times 10^{-2}$</td>
<td>$4$</td>
<td>$3.1 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 5. Values of $A_{ER}$ determined for two transducers of diameter 1.27 cm, and frequencies of 1 MHz and 2.25 MHz, and two transducers of diameter 2.54 cm, and frequencies of 1 MHz and 3.5 MHz. Type A, type B and expanded uncertainties are also presented.

<table>
<thead>
<tr>
<th></th>
<th>Transducers of 1.27 cm</th>
<th>Transducers of 2.54 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0 MHz</td>
<td>2.25 MHz</td>
</tr>
<tr>
<td>$A_{ER}$ (1)/cm$^2$</td>
<td>1.18</td>
<td>1.15</td>
</tr>
<tr>
<td>$A_{ER}$ (2)/cm$^2$</td>
<td>1.19</td>
<td>1.18</td>
</tr>
<tr>
<td>$A_{ER}$ (3)/cm$^2$</td>
<td>1.18</td>
<td>1.16</td>
</tr>
<tr>
<td>$A_{ER}$ (4)/cm$^2$</td>
<td>1.18</td>
<td>1.12</td>
</tr>
<tr>
<td>$A_{ER}$ (mean)/cm$^2$</td>
<td>1.181</td>
<td><strong>1.158</strong></td>
</tr>
<tr>
<td>$\mu_{type}$ /cm$^2$</td>
<td>$2.50 \times 10^{-3}$</td>
<td>$1.25 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\mu_{type}$ /cm$^2$</td>
<td>$3.14 \times 10^{-2}$</td>
<td>$3.66 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\mu_{combined}$ /cm$^2$</td>
<td>$3.15 \times 10^{-2}$</td>
<td>$3.86 \times 10^{-2}$</td>
</tr>
<tr>
<td>Coverage factor (95%)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$u_{expanded}$/cm$^2$</td>
<td>$6.3 \times 10^{-2}$</td>
<td>$7.9 \times 10^{-2}$</td>
</tr>
<tr>
<td>$u_{expanded}$/%</td>
<td>5.3</td>
<td><strong>6.8</strong></td>
</tr>
</tbody>
</table>

Table 6. Values of transducer effective radius ($a_t$) and effective area ($A_E$), calculated based on the determination of $Z_N$.

<table>
<thead>
<tr>
<th></th>
<th>Tx. 1 MHz</th>
<th>Tx. 2.25 MHz</th>
<th>Tx. 1 MHz</th>
<th>Tx. 3.5 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varnothing = 1.27$ cm</td>
<td>$\varnothing = 1.27$ cm</td>
<td>$\varnothing = 2.54$ cm</td>
<td>$\varnothing = 2.54$ cm</td>
</tr>
<tr>
<td>$Z_N$/cm</td>
<td>2.64</td>
<td>5.70</td>
<td>9.30</td>
<td>351.0</td>
</tr>
<tr>
<td>$a_t$/cm</td>
<td>0.63</td>
<td>0.61</td>
<td>1.17</td>
<td>1.22</td>
</tr>
<tr>
<td>$A_E$/cm$^2$</td>
<td>1.23</td>
<td>1.18</td>
<td>4.32</td>
<td>4.66</td>
</tr>
<tr>
<td>$A_{ER}$/cm$^2$</td>
<td>$1.181 \pm 0.063$</td>
<td>$1.158 \pm 0.079$</td>
<td>$4.22 \pm 0.18$</td>
<td>$4.70 \pm 0.17$</td>
</tr>
</tbody>
</table>

Results indicate coherence between the $A_{ER}$ and $A_E$ estimations using the presented protocol and measuring system, in that $(A_{ER} - 2u_{A_{ER}}) < A_E < (A_{ER} + 2u_{A_{ER}})$. One observes in table 5 that the type B uncertainties were larger than the type A uncertainties for all four transducers used. The type A uncertainties were of the same magnitude (absolute value) for the two head diameters tested (1.27 cm and 2.54 cm). On the other hand, and as expected, the type B uncertainties increased in proportion to the transducer diameter. These are the main reasons for the lower relative (%) expanded uncertainty for the 2.54 cm transducer. Despite the fact that a different approach was used in [13, 15, 16] to assess the uncertainty in $A_{ER}$, the present work finds similar values of expanded uncertainty (4% to 7%). With these few data produced in this work, it was not possible to infer any relation to uncertainty and transducer nominal frequency for $A_{ER}$.
of diameter 2.54 cm, and frequencies of 1 MHz and 3.5 MHz. Type A, type B and expanded uncertainties are also presented.

Values of $R_{BN}$ for two transducers of diameter 1.27 cm, and frequencies of 1 MHz and 2.25 MHz, and two transducers of diameter 2.54 cm, and frequencies of 1 MHz and 3.5 MHz. Type A, type B and expanded uncertainties are also presented.

<table>
<thead>
<tr>
<th></th>
<th>Transducers of 1.27 cm</th>
<th></th>
<th>Transducers of 2.54 cm</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0 MHz</td>
<td>2.25 MHz</td>
<td>1.0 MHz</td>
<td>3.5 MHz</td>
</tr>
<tr>
<td>$R_{BN}$ (1)</td>
<td>3.29</td>
<td>2.85</td>
<td>3.31</td>
<td>3.42</td>
</tr>
<tr>
<td>$R_{BN}$ (2)</td>
<td>3.20</td>
<td>2.71</td>
<td>3.30</td>
<td>3.57</td>
</tr>
<tr>
<td>$R_{BN}$ (3)</td>
<td>3.25</td>
<td>2.69</td>
<td>3.20</td>
<td>3.52</td>
</tr>
<tr>
<td>$R_{BN}$ (4)</td>
<td>3.22</td>
<td>2.77</td>
<td>3.19</td>
<td>3.52</td>
</tr>
<tr>
<td>$R_{BN}$ (mean)</td>
<td>3.24</td>
<td>2.76</td>
<td>3.25</td>
<td>3.51</td>
</tr>
<tr>
<td>$u_{Type A}$</td>
<td>$2.01 \times 10^{-2}$</td>
<td>$3.43 \times 10^{-2}$</td>
<td>$3.06 \times 10^{-2}$</td>
<td>$3.33 \times 10^{-2}$</td>
</tr>
<tr>
<td>$u_{Type B}$</td>
<td>$2.25 \times 10^{-1}$</td>
<td>$2.01 \times 10^{-1}$</td>
<td>$2.00 \times 10^{-1}$</td>
<td>$2.19 \times 10^{-1}$</td>
</tr>
<tr>
<td>$u_{Combined}$</td>
<td>$2.26 \times 10^{-1}$</td>
<td>$2.03 \times 10^{-1}$</td>
<td>$2.02 \times 10^{-1}$</td>
<td>$2.21 \times 10^{-1}$</td>
</tr>
<tr>
<td>Coverage factor (95%)</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$u_{Expanded}$</td>
<td>$4.6 \times 10^{-1}$</td>
<td>$4.1 \times 10^{-1}$</td>
<td>$4.1 \times 10^{-1}$</td>
<td>$4.5 \times 10^{-1}$</td>
</tr>
<tr>
<td>$u_{Expanded/%}$</td>
<td>14.2</td>
<td>14.9</td>
<td>12.6</td>
<td>12.8</td>
</tr>
</tbody>
</table>

### Figure 2

Results obtained for 2.25 MHz transducer ($\phi = 1.27$ cm), during test 2: (a) plane mapped at 0.3 cm from the transducer face; (b) plane mapped at $Z_N$. The $A_{ER}$ value obtained at the plane in (a) is 1.18 cm$^2$ (table 5).

### Figure 3

Results obtained for 3.5 MHz transducer ($\phi = 2.54$ cm), during test 2: (a) plane mapped at 0.3 cm from the transducer face; (b) plane mapped at $Z_N$. The $A_{ER}$ value obtained at the plane in (a) is 4.67 cm$^2$ (table 5).

Table 7. Values of $R_{BN}$ determined for two transducers of diameter 1.27 cm, and frequencies of 1 MHz and 2.25 MHz, and two transducers of diameter 2.54 cm, and frequencies of 1 MHz and 3.5 MHz. Type A, type B and expanded uncertainties are also presented.

### 10. Conclusion

The ultrasonic pressure field mapping system developed at Labus (Inmetro) can be used to carry out mappings and calculations needed to determine the parameters related to the ultrasonic beam of transducers used in physiotherapy, based on IEC 61689:2007. This new system represents an improvement compared with that presented in previous works [9, 10]. The expanded uncertainties, of less than 7% ($A_{ER}$) and 15% ($R_{BN}$) (95% confidence level), achieved using transducers of different diameters (1.27 cm and 2.54 cm) and frequencies (1 MHz to 3.5 MHz) are lower than those reported in [4] ($u_{A_{ER}} < \pm 10\%$ and $u_{R_{BN}} < \pm 15\%$). Hence, we conclude that Labus is prepared to estimate $A_{ER}$ and $R_{BN}$ and their respective uncertainties in accordance with IEC 61689:2007.
Acknowledgments

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References

[4] IEC 61689:2007 Ultrasonics—Physiotherapy systems—Field specifications and methods of measurement in the frequency range 0.5 MHz to 5 MHz.